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A collection of metric Mahler measures

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Abstract. Let $M(\alpha)$ denote the Mahler measure of the algebraic number α. In a recent paper, Dubickas and Smyth constructed a metric version of the Mahler measure on the multiplicative group of algebraic numbers. Later, Fili and the author used similar techniques to study a non-Archimedean version. We show how to generalize the above constructions in order to associate, to each point in $(0, \infty]$, a metric version M_x of the Mahler measure, each having a triangle inequality of a different strength. We are able to compute $M_x(\alpha)$ for sufficiently small x, identifying, in the process, a function $\bar{M}$ with certain minimality properties. Further, we show that the map $x \mapsto M_x(\alpha)$ defines a continuous function on the positive real numbers.

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