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# Primes of the form $X^2 + nY^2$ in function fields and Drinfeld modules

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**Abstract.** Let $n$ be a square-free polynomial over $\mathbb{F}_q$, where $q$ is an odd prime power. In this paper, we determine which irreducible polynomials $p$ in $\mathbb{F}_q[x]$ can be represented in the form $X^2 + nY^2$ with $X, Y \in \mathbb{F}_q[x]$. We restrict ourselves to the case where $X^2 + nY^2$ is anisotropic at $\infty$. As in the classical case over $\mathbb{Z}$ discussed in [2], the representability of $p$ by the quadratic form $X^2 + nY^2$ is governed by conditions coming from class field theory. A necessary (and almost sufficient) condition is that the ideal generated by $p$ splits completely in the Hilbert class field $\mathcal{H}$ of $\mathcal{K} = \mathbb{F}_q(x, \sqrt{-n})$ (for the appropriate notion of Hilbert class field in this context). In order to get explicit conditions for $p$ to be of the form $X^2 + nY^2$, we use the theory of sgn-normalized rank-one Drinfeld modules. We present an algorithm to construct a generating polynomial of $\mathcal{H}/\mathcal{K}$. This algorithm generalizes to all situations an algorithm of D.S. Dummit and D. Hayes for the case where $-n$ is monic of odd degree.