

## The Lang-Trotter conjecture on average

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**Abstract.** For an elliptic curve  $E$  over  $\mathbb{Q}$  and an integer  $r$  let  $\pi_E^r(x)$  be the number of primes  $p \leq x$  of good reduction such that the trace of the Frobenius morphism of  $E/\mathbb{F}_p$  equals  $r$ . We consider the quantity  $\pi_E^r(x)$  on average over certain sets of elliptic curves. More in particular, we establish the following: If  $A, B > x^{1/2+\varepsilon}$  and  $AB > x^{3/2+\varepsilon}$ , then the arithmetic mean of  $\pi_E^r(x)$  over all elliptic curves  $E : y^2 = x^3 + ax + b$  with  $a, b \in \mathbb{Z}$ ,  $|a| \leq A$  and  $|b| \leq B$  is  $\sim C_r \sqrt{x} / \log x$ , where  $C_r$  is some constant depending on  $r$ . This improves a result of C. David and F. Pappalardi. Moreover, we establish an “almost-all” result on  $\pi_E^r(x)$ .

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