

# Deligne's notes on Nagata compactifications

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**Abstract.** We provide a proof of Nagata's compactification theorem: any separated map of finite type between quasi-compact and quasi-separated schemes (e.g., noetherian schemes) factors as an open immersion followed by a proper morphism. This is a detailed exposition of private notes of Deligne that translate Nagata's method into modern terms, and includes some applications of general interest in the theory of rational maps, such as refined versions of Chow's Lemma and the elimination of indeterminacies in a rational map, as well as a blow-up characterization of when a proper morphism (to a rather general base scheme) is birational.

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