

A theta function identity and the Eisenstein series on $\Gamma_0(5)$

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Abstract. We prove an identity connecting a theta function and a sum of Eisenstein series by using the complex variable theory of elliptic functions, which contains as a special case a famous identity of Ramanujan connected with partitions modulus 5. This identity allows us to develop a theory for the Eisenstein series on the congruence subgroup $\Gamma_0(5)$. Combining this identity with two identities in Ramanujan's lost notebook, a curious Eisenstein series identity related to the Rogers-Ramanujan continued fraction is derived.

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