

Analytic properties of residual Eisenstein series, I

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Abstract. We partially generalize the results of [16] on the poles of degenerate, Siegel-parabolic Eisenstein series to residual-data Eisenstein series. In particular, for a, b integers greater than 1, we show that poles of the Eisenstein series induced from the Speh representation $\Delta(\tau, b)$ on the Levi GL_{ab} of Sp_{2ab} are located in the ‘segment’ of half integers X_b between a ‘right endpoint’ and its negative, inclusive of endpoints. The right endpoint is $\pm b/2$, or $(b-1)/2$, depending on the analytic properties of the automorphic L -functions attached to τ . We study the automorphic forms $\Phi_{\frac{1}{2}}^{(b)}$ obtained as residues at the points $s_{\frac{1}{2}}^{(b)}$ (defined precisely in the paper) by calculating their cuspidal exponents in certain cases. In the case of the ‘endpoint’ $s_0^{(b)}$ and ‘first interior point’ $s_1^{(b)}$ in the segment of singularity points, we are able to determine a set containing *all possible* cuspidal exponents of $\Phi_0^{(b)}$ and $\Phi_1^{(b)}$ precisely for all a and b . In these cases, we use the result of the calculation to deduce that the residual automorphic forms lie in $L^2(G(k)\backslash G(\mathbf{A}))$. In a more precise sense, our result establishes a relationship between, on the one hand, the actually occurring cuspidal exponents of $\Phi_{\frac{1}{2}}^{(b)}$, residues at interior points which lie to the right of the origin, and, on the other hand, the ‘analytic properties’ of the original residual-data Eisenstein series at the origin.

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