

The sign of an elliptic divisibility sequence

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Abstract. An elliptic divisibility sequence (EDS) is a sequence of integers $(W_n)_{n \geq 0}$ generated by the nonlinear recursion satisfied by the division polynomials of an elliptic curve. We give a formula for the sign of W_n for unbounded nonsingular EDS, a typical case being $\text{Sign}(W_n) = (-1)^{\lfloor n\beta \rfloor}$ for an irrational number $\beta \in \mathbb{R}$. As an application, we show that the associated sequence of absolute values $(|W_n|)$ cannot be realized as the fixed point counting sequence of any abstract dynamical system.

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