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Conservation and invariance properties of submarkovian semigroups

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Abstract. Let $\mathcal{E}$ be a Dirichlet form on $L_2(X)$ and Ω an open subset of X. Then one can define Dirichlet forms $\mathcal{E}_D$, or $\mathcal{E}_N$, corresponding to $\mathcal{E}$ but with Dirichlet, or Neumann, boundary conditions imposed on the boundary $\partial\Omega$ of Ω. If S, S^D and S^N are the associated submarkovian semigroups we prove, under general assumptions of regularity and locality, that $S_t\varphi = S^D_t\varphi$ for all $\varphi \in L_2(\Omega)$ and $t > 0$ if and only if the capacity $\mathrm{cap}_\Omega(\partial\Omega)$ of $\partial\Omega$ relative to Ω is zero. Moreover, if S is conservative, i.e. stochastically complete, then $\mathrm{cap}_\Omega(\partial\Omega) = 0$ if and only if S^D is conservative on $L_2(\Omega)$. Under slightly more stringent assumptions we also prove that the vanishing of the relative capacity is equivalent to $S^D_t\varphi = S^N_t\varphi$ for all $\varphi \in L_2(\Omega)$ and $t > 0$.

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