

# Cuntz's $ax + b$ -semigroup $C^*$ -algebra over $\mathbb{N}$ and product system $C^*$ -algebras

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**Abstract.** We investigate  $C^*$ -algebras associated with row-finite topological higher-rank graphs with no source, which are based on product system  $C^*$ -algebras. We prove the Cuntz–Krieger uniqueness theorem, and provide the condition of simplicity and purely infiniteness of our algebras. We give examples of non-discrete topological higher-rank graphs whose  $C^*$ -algebras contain Cuntz's  $ax + b$ -semigroup  $C^*$ -algebra over  $\mathbb{N}$ .

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